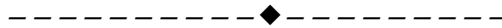


Availability Analysis of a System Having Two Units in Series Configuration with Controller and Human Failure under Different Repair Policies

V. V. Singh, Dilip Kumar Rawal

Abstract - This paper deals with the availability analysis of a complex system that consists of two subsystems namely subsystem 1 and subsystem 2. Subsystem 1 is working under k-out of -n good policy and subsystem 2 has two identical units in parallel configuration. Controller for proper functioning controls the subsystems 1. All failure rates are constant and follow exponential distribution but repairs follow general and Gumbel-Hougaard family copula distribution. The system is analyzed by supplementary technique by evaluating various measures of reliability such as state transition probability, MTTF, etc. Some computations are taken as special cases by evaluating availability of system and profit analysis.

Keywords : Controller, Gumbel-Hougaard family copula, human failure, MTTF, k-out of n policy, supplementary variable, profit function.



INTRODUCTION:

Many authors including [1, 6, 8] have discussed reliability of

Complex systems by taking various failure and one repair policy. By thinking about present scenario and complexity of advanced technology and modern demand of electronic equipments one needs to study of a controller, which is used in, various electronic devices and systems.

In this paper the authors have considered a complex system, which consists of two subsystems 1 and subsystem 2. The subsystem 1 follows (k-out of -n good) policy, the subsystem 2 has two identical units in parallel configuration. Both subsystems are connected in series. The subsystem-1 controlled by a controller. The system can fail in following situations: (1) more than k units of subsystem 1 has failed but both units of subsystem 2 are in good working condition, (2) Human failure occurs in system, (3) Controller of subsystem 1 fails, (4) Both units of subsystem 2 fail. The system will be in minor partial failure in following situations: (1) All units of subsystem 1 are good and one unit of subsystem 2 has failed, (2) At least k units of subsystem 1 are good and one unit of subsystem 2 has failed.

Many authors have considered reliability and MTTF of a complex system, with different types of failures and one type of repair. However, they did not consider one of the important aspects of repair between two transition states i.e. how system

will be behaving when there are two different types of repair possible between two adjacent states, which seems to be possible in many engineering systems. When this possibility exists, reliability of the system can be analyzed with the help of copula [7]. The authors [10] have discussed the availability of a system having three units under pre-emptive resume repair policy using copula in deliberately failure state. Therefore, in contrast to the earlier models, here the author has considered a model in which he tried to address the problem where two different repair facilities are available between adjacent states i.e. the initial state and complete failed states. All failure rates are assumed to follow negative exponential distribution. The repairs follow general and Gumbel-Hougaard family copula distributions. In present paper, S_0 is state where the system is in good working condition. S_1, S_3, S_4 are states where the system is in partial or degraded mode and states S_2, S_5, S_6, S_7 , and S_8 are states where the system is in completely failure mode. When the system is in degraded mode, the general repaired is employed but whenever the system is in completely failure mode, the system is repaired by Gumbel-Hougaard family copula [7]. The system is analyzed by supplementary variable technique and various measures of reliability have been discussed and some particular cases are also taken to highlight the result. The results are demonstrated by graphs and conclusions are drawn by graphs.

Author Dr.V.V.Singh is a professor in the department of mathematics at DIT School of engineering Greater Noida (Gautam Budha Nagar) (India)
Email:singh_vijayvir@yahoo.com ,Ph.01202390983

author Dilip Kumar Rawal is a research scholar in mathematics department at Mewar university Chittorgarh Raj.(India)
Email:diliprawa05@gmail.com

State description of model	
State	State Description
S_0	All units of sub system 1 and 2 are good working condition.
S_1	k units of sub system -1 are in good working condition.
S_2	System has completely fail due to failure of (k+i) units of subsystem-1.
S_3	One unit of sub system -2 has failed and the system is in degraded state.
S_4	All units of sub system -1 are good and one unit of sub system has failed.
S_5	k units of sub system -1 are good but both units of sub system -2 has failed. The system is completely failed state. (1) (2)
S_6	All unit of sub system 1 are good but both units of sub system -2 have failed. The system is completely failed state. (3) (4)
S_7	System has been failed due to failure of controller in sub system -1. (5)
S_8	System is completely failed due to human failure. (6)
NOTATIONS:	
λ_1	Failure rate of 1 unit in sub system -1 (7)
λ_1 / λ_2	Failure rates of subsystem 1 such that at most k unit /more than k units failed during operational mode. (8)
λ_n / λ_c	Failure rates due to human failure/controller of subsystem-1.
λ	Failure rates of sub system -2.
$\phi(x)$	Minor repair rates for state S_1, S_3 and S_4 .
$C_\theta(u_1, u_2(x))$	$C_\theta(u_1, u_2(x)) = \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta}$, where, $u_1 = \phi(x)$, and $u_2 = e^x$, where θ is a parameter.
$P_i(t)$	Probability that the system is in state $S_i, i=0$
λ_1	Failure rate of 1 unit in sub system -1.
λ_1 / λ_2	Failure rates of subsystem 1 such that at most k unit /more than k units failed during operational mode.

$E_p(t)$:	Expected profit during the interval $[0,t]$. K_1, K_2 Revenue per unit time and service cost in interval $[0, t)$.
$P(s)$	Laplace transform of $P(t)$
$P_i(x,t)$:	Probability that system is in state $i = 1, 2, 3, 4, 5, 6, 7, 8$, System is running under repair and elapsed repair time is x, t .

ASSUMPTION:

The following assumptions are taken throughout the discussion of model.

- (1) Initially the system is in S_0 state and all units of subsystem-1 and 2 are in good working condition.
- (2) The sub system 1 works successfully till at least k-units of it is in good working condition.
- (3) Subsystem 1 fails if more than k units fail. Sub system 2 work successfully if at least one unit is good.
- (4) Subsystem 2 may repair when one unit fails or both unit fail or controller fails.
- (5) All failure rates are constant and follow exponential distribution.
- (6) Minor partial failure is repaired by general time distribution.
- (7) Human failure /complete failure system need fast repairing (Gumbel-Hougaard) family copula.
- (8) Repaired system works like a new and repair did not damage anything.

Transition of Model:

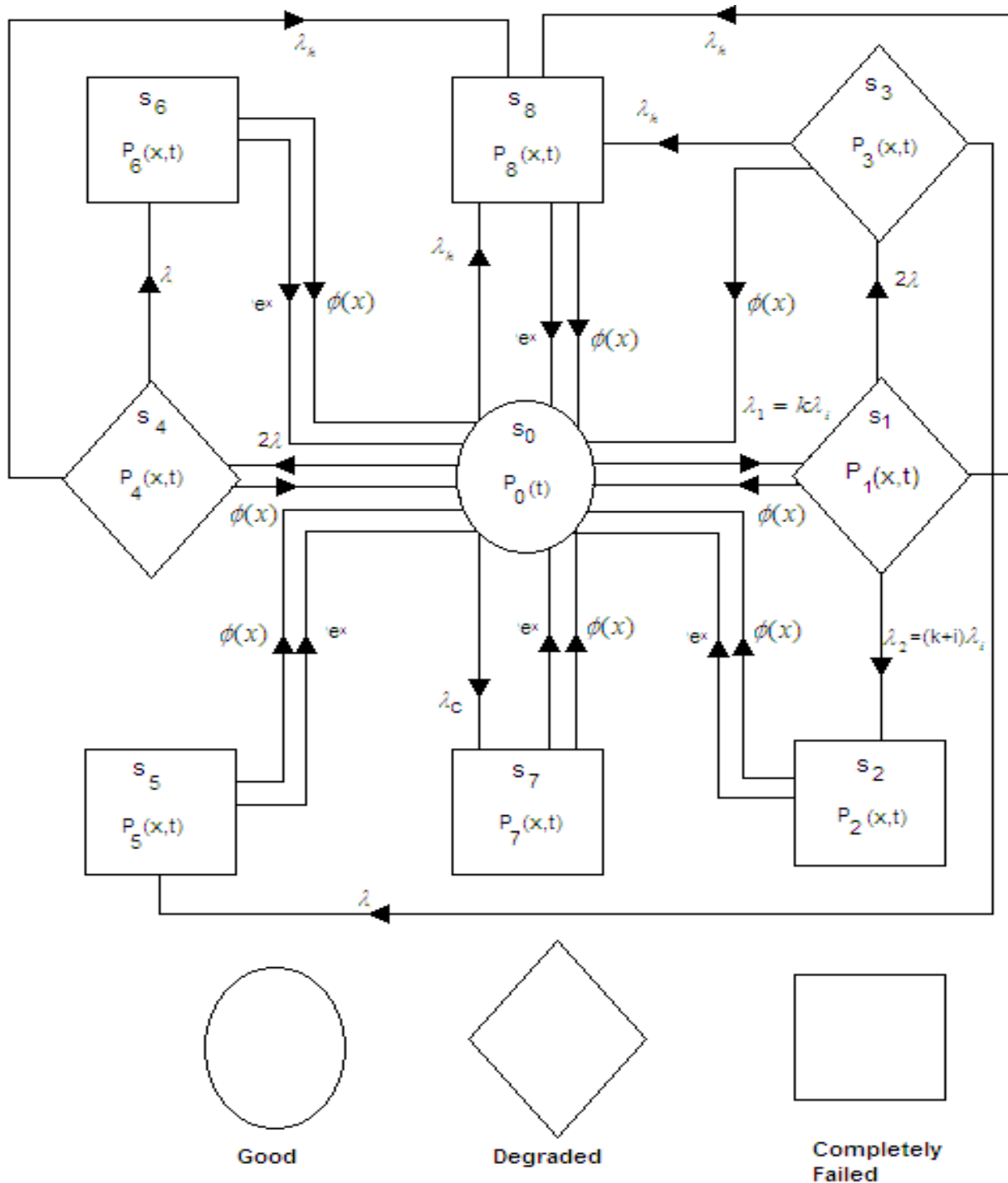


Fig.1 State transition Diagram Of Model

FORMULATION OF MATHEMATICAL MODEL:

By probability of considerations and continuity arguments, we can obtain the following set of difference differential equations governing the present mathematical model.

$$\left[\frac{\partial}{\partial t} + \lambda_1 + \lambda_c + \lambda_h + 2\lambda \right] P_0(t) = \int_0^\infty \phi(x) P_1(x,t) dx + \int_0^\infty \phi(x) P_4(x,t) dx + \int_0^\infty \phi(x) P_3(x,t) dx + \int_0^\infty \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta} P_7(x,t) dx + \int_0^\infty \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta} P_6(x,t) dx + \int_0^\infty \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta} P_8(x,t) dx + \int_0^\infty \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta} P_5(x,t) dx + \int_0^\infty \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta} P_2(x,t) dx \dots (1)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_2 + \lambda_c + \lambda_h + 2\lambda + \phi(x) \right] P_1(x,t) = 0 \dots (2)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta} \right] P_2(x,t) = 0 \dots (3)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_c + \lambda_h + \lambda + \phi(x) \right] P_3(x,t) = 0 \dots (4)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_c + \lambda_h + \lambda + \phi(x) \right] P_4(x,t) = 0 \dots (5)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta} \right] P_5(x,t) = 0 \dots (6)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta} \right] P_6(x,t) = 0 \dots (7)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta} \right] P_7(x,t) = 0 \dots (8)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta} \right] P_8(x,t) = 0 \dots (9)$$

BOUNDARY CONDITIONS:

$$P_1(0,t) = \lambda_1 P_0(t) \dots (10)$$

$$P_2(0,t) = \lambda_1 \lambda_2 P_0(t) \dots (11)$$

$$P_3(0,t) = 2\lambda_1 \lambda P_0(t) \dots (12)$$

$$P_4(0,t) = 2\lambda P_0(t) \dots (13)$$

$$P_5(0,t) = 2\lambda^2 \lambda_1 P_0(t) \dots (14)$$

$$P_6(0,t) = 2\lambda^2 P_0(t) \dots (15)$$

$$P_7(0,t) = \lambda_c (1 + \lambda_1) (1 + 2\lambda) P_0(t) \dots (16)$$

$$P_8(0,t) = \lambda_h (1 + \lambda_1) (1 + 2\lambda) P_0(t) \dots (17)$$

INITIALS CONDITIONS:

$$P_0(0) = 1 \text{ and other state probabilities are zero at } t = 0 \dots (18)$$

SOLUTION OF THE MODEL:

Taking Laplace transformation of equations (1)-(17) and using equation (18), we obtain.

$$\left[s + \lambda_1 + \lambda_c + 2\lambda + \lambda_h \right] \bar{P}_0(s) = \left[1 + \int_0^\infty \bar{P}_1(x,s) \phi(x) dx + \int_0^\infty \bar{P}_7(x,s) \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta} dx + \int_0^\infty \bar{P}_4(x,s) \phi(x) dx + \int_0^\infty \bar{P}_6(x,s) \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta} dx + \int_0^\infty \bar{P}_8(x,s) \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta} dx + \int_0^\infty \bar{P}_2(x,s) \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta} dx + \int_0^\infty \bar{P}_5(x,s) \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta} dx + \int_0^\infty \bar{P}_3(x,s) \phi(x) dx \right] \dots (19)$$

$$\left[s + \frac{\partial}{\partial x} + \lambda_2 + 2\lambda + \lambda_c + \lambda_h + \phi(x) \right] \bar{P}_1(x, s) = 0 \dots (20)$$

$$\left[s + \frac{\partial}{\partial x} + \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta} \right] \bar{P}_5(x, s) = 0 \dots (24)$$

$$\left[s + \frac{\partial}{\partial x} + \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta} \right] \bar{P}_2(x, s) = 0 \dots (21)$$

$$\left[s + \frac{\partial}{\partial x} + \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta} \right] \bar{P}_6(x, s) = 0 \dots (25)$$

$$\left[s + \frac{\partial}{\partial x} + \lambda + \lambda_c + \lambda_h + \phi(x) \right] \bar{P}_3(x, s) = 0 \dots (22)$$

$$\left[s + \frac{\partial}{\partial x} + \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta} \right] \bar{P}_7(x, s) = 0 \dots (26)$$

$$\left[s + \frac{\partial}{\partial x} + \lambda + \lambda_c + \lambda_h + \phi(x) \right] \bar{P}_4(x, s) = 0 \dots (23)$$

$$\left[s + \frac{\partial}{\partial x} + \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta} \right] \bar{P}_8(x, s) = 0 \dots (27)$$

one may get,

$$\bar{P}_1(0, s) = \lambda_1 \bar{P}_0(s) \dots (28)$$

$$\bar{P}_0(s) = \frac{1}{D(s)} \dots (36)$$

$$\bar{P}_2(0, s) = \lambda_1 \lambda_2 \bar{P}_0(0, s) \dots (29)$$

$$\bar{P}_1(s) = \frac{\lambda_1}{D(s)} \frac{(1 - S_\phi(s + \lambda_2 + 2\lambda + \lambda_c + \lambda_h))}{(s + \lambda_2 + 2\lambda + \lambda_c + \lambda_h)} \dots (37)$$

$$\bar{P}_3(0, s) = 2\lambda_1 \lambda \bar{P}_0(0, s) \dots (30)$$

$$\bar{P}_2(s) = \frac{\lambda_1 \lambda_2}{D(s)} \frac{(1 - S_{\exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta}}(s))}{(s)} \dots (38)$$

$$\bar{P}_4(0, s) = 2\lambda^2 \bar{P}_0(s) \dots (31)$$

$$\bar{P}_3(s) = \frac{2\lambda_1 \lambda}{D(s)} \frac{(1 - S_\phi(s + \lambda + \lambda_c + \lambda_h))}{(s + \lambda + \lambda_c + \lambda_h)} \dots (39)$$

$$\bar{P}_5(0, s) = 2\lambda_1 \lambda^2 \bar{P}_0(s) \dots (32)$$

$$\bar{P}_4(s) = \frac{2\lambda}{D(s)} \frac{(1 - S_\phi(s + \lambda + \lambda_c + \lambda_h))}{(s + \lambda + \lambda_c + \lambda_h)} \dots (40)$$

$$\bar{P}_6(0, s) = 2\lambda^2 \bar{P}_0(s) \dots (33)$$

$$\bar{P}_7(0, s) = \lambda_c (1 + \lambda_1) (1 + 2\lambda) \bar{P}_0(s) \dots (34)$$

$$\bar{P}_5(s) = \frac{2\lambda_1 \lambda^2}{D(s)} \frac{(1 - S_{\exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta}}(s))}{(s)} \dots (41)$$

$$\bar{P}_8(0, s) = \lambda_h (1 + \lambda_1) (1 + 2\lambda) \bar{P}_0(s) \dots (35)$$

Solving (19)-(27), with help of equations (28) to (35)

$$\bar{P}_6(s) = \frac{2\lambda^2}{D(s)} \frac{(1 - S_{\exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta}}(s))}{(s)} \dots (42)$$

$$\bar{P}_8(s) = \frac{\lambda_h (1 + \lambda_1) (1 + 2\lambda)}{D(s)} \frac{(1 - S_{\exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta}}(s))}{(s)} \dots (44)$$

$$\bar{P}_7(s) = \frac{\lambda_c (1 + \lambda_1) (1 + 2\lambda)}{D(s)} \frac{(1 - S_{\exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta}}(s))}{(s)} \dots (43)$$

$$\begin{aligned}
 D(s) = & s + \lambda_1 + \lambda_c + \lambda_h + 2\lambda - \{\lambda_1 S_\phi(s + \lambda_2 + 2\lambda + \lambda_c + \lambda_h) \\
 & + \lambda_c(1 + \lambda_1)(1 + 2\lambda) S_{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}(s) + \\
 & \lambda_h(1 + \lambda_1)(1 + 2\lambda) S_{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}(s) + \\
 & 2\lambda S_\phi(s + \lambda + \lambda_c + \lambda_h) + \lambda_1 \lambda_2 S_{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}(s) + \\
 & 2\lambda_1 \lambda^2 S_{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}(s) \} \dots (45)
 \end{aligned}$$

The Laplace transformations of the probabilities that the system is in up (i.e. either good or degraded state) and failed state at any time are as follows;

$$\bar{P}_{up}(s) = \bar{P}_0(s) + \bar{P}_1(s) + \bar{P}_4(s) + \bar{P}_3(s)$$

$$= \frac{1}{D(s)} \left[\begin{aligned} & 1 + \lambda_1 \frac{(1 - S_\phi(s + \lambda_2 + 2\lambda + \lambda_c + \lambda_h))}{(s + \lambda_2 + 2\lambda + \lambda_c + \lambda_h)} + \\ & 2\lambda \frac{(1 - S_\phi(s + \lambda + \lambda_c + \lambda_h))}{(s + \lambda + \lambda_c + \lambda_h)} + \\ & 2\lambda_1 \lambda \frac{(1 - S_\phi(s + \lambda + \lambda_c + \lambda_h))}{(s + \lambda + \lambda_c + \lambda_h)} \end{aligned} \right] \dots(46)$$

PARTICULAR CASES:

When repair follows exponential distribution, setting

$$\bar{S}_{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}(s) = \frac{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}{s + \exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}$$

$$\bar{S}_\phi(s) = \frac{\phi}{s + \phi}, \text{ in equation (46) and}$$

(1) Setting the values of different parameters as

$$\lambda_1 = 0.05, \lambda_2 = 0.06, \lambda_c = 0.01, \lambda_h = 0.01,$$

$$\lambda = 0.005, \phi = 1, \theta = 1 \text{ and } x = 1,$$

$$\lambda_1 = k\lambda_i, \lambda_2 = (k + i)\lambda_i,$$

in (46), then taking the inverse Laplace transform, one can obtain,

$$\begin{aligned}
 P_{up}(t) = & -0.012358e^{(-2.12547t)} - 0.0051717e^{(-1.1505t)} - \\
 & 0.000414e^{(-1.0559t)} + 0.99285e^{(-0.0024957t)} \dots(48)
 \end{aligned}$$

(II) Setting the values of different parameters as

$$\begin{aligned}
 \bar{P}_{failed}(s) = & \bar{P}_2(s) + \bar{P}_5(s) + \bar{P}_6(s) + \bar{P}_7(s) + \bar{P}_8(s) \\
 = & \frac{\lambda_1}{D(s)} \frac{(1 - S_\phi(s + \lambda_2 + 2\lambda + \lambda_c + \lambda_h))}{(s + \lambda_2 + 2\lambda + \lambda_c + \lambda_h)} + \\
 & \frac{2\lambda_1 \lambda^2 (1 - S_{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}(s))}{D(s) (s)} + \\
 & \frac{2\lambda^2 (1 - S_{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}(s))}{D(s) (s)} + \\
 & \frac{\lambda_c(1 + \lambda_1)(1 + 2\lambda) (1 - S_{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}(s))}{D(s) (s)} + \\
 & \frac{\lambda_h(1 + \lambda_1)(1 + 2\lambda) (1 - S_{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}(s))}{D(s) (s)} \dots(47)
 \end{aligned}$$

$$\lambda_1 = 0.06, \lambda_2 = 0.07, \lambda_c = 0.01, \lambda_h = 0.01, \lambda = 0.005, \phi = 1, \theta = 1 \text{ and } x = 1$$

$$\begin{aligned}
 P_{up}(t) = & -0.01296e^{(-2.7541t)} - 0.0064844e^{(-1.1688t)} - \\
 & 0.000025854e^{(-1.0560t)} + 0.99355e^{(-0.002325t)} \dots(49)
 \end{aligned}$$

For, $t = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90 \dots$ one may get different values of $P_{up}(t)$ as shown in Table 1. If there are n identical units in subsystem-1 in parallel configuration and two units in parallel configuration in subsystem 2 and $\lambda_1 = k\lambda_i, \lambda_2 = (k + i)\lambda_i,$

Time (t)	$P_{up}(t)$ $\lambda_1 = 0.05,$ $\lambda_2 = 0.06,$	$P_{up}(t)$ $\lambda_1 = 0.06,$ $\lambda_2 = 0.07,$	$P_{up}(t)$ $\lambda_1 = 0.07,$ $\lambda_2 = 0.08,$	$P_{up}(t)$ $\lambda_1 = 0.08,$ $\lambda_2 = 0.09,$	$P_{up}(t)$ $\lambda_1 = 0.09,$ $\lambda_2 = 0.10,$
0	1.0000	1.000	1.000	1.000	1.000
10	0.9684	0.9707	0.9734	0.9767	0.9804
20	0.9445	0.9484	0.9531	0.9587	0.9652
30	0.9212	0.9266	0.9331	0.9411	0.9503
40	0.8985	0.9053	0.9137	0.9237	0.9356
50	0.8764	0.8845	0.8946	0.9067	0.9212
60	0.8548	0.8641	0.8757	0.8900	0.9069
70	0.8337	0.8443	0.8576	0.8736	0.8926
80	0.8132	0.8249	0.8396	0.8575	0.8791
90	0.7931	0.8060	0.8221	0.8417	0.8655

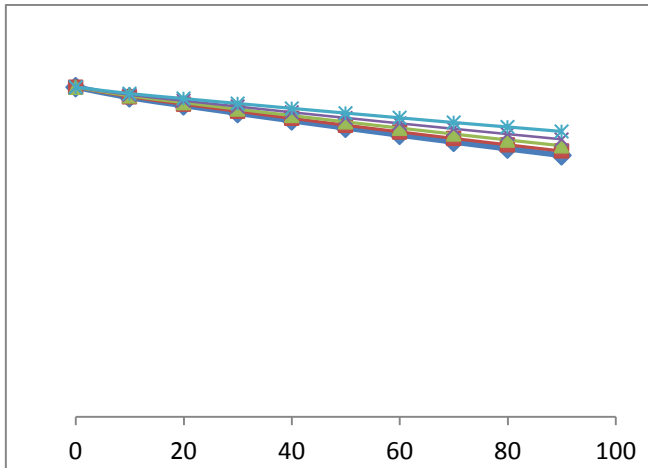


Fig.1 Time V/S Availability

Mean Time To Failure (M.T.T.F.):

Setting

$$\bar{S}_{\exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta}}(s) = \frac{\exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta}}{s + \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta}}$$

$\bar{S}_\phi(s) = \frac{\phi}{s + \phi}$, and taking all repairs to zero in equation (46) and taking limit as s tend to zero one

can obtain the MTTF as;

$$M.T.T.F. = \lim_{s \rightarrow 0} \bar{P}_{up}(s) \frac{1}{(\lambda_1 + \lambda_c + \lambda_h + 2\lambda)} \dots(50)$$

Setting $\lambda_c=0.01$, $\lambda_h=0.01$, $\lambda=0.005$ and varying λ_1 as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10 in (50), one may obtain Table2. Which demonstrates variation of MTTF with respect to failure rates.

Setting $\lambda_1=0.05$, $\lambda_h=0.01$, $\lambda=0.005$ and varying λ_c as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10 in (50), one may obtain Table2. Which demonstrates variation of MTTF with respect to λ_c .

Setting $\lambda_1=0.05$, $\lambda_c=0.01$, λ and varying λ_h as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10 in (50), one may obtain Table2. Which demonstrates variation of MTTF with respect to failure rates λ_c .

Setting $\lambda_1=0.05$, $\lambda_c=0.01$, λ_h and varying λ as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10 in

(50), one may obtain Table2. Which demonstrates variation of MTTF with respect to failure rates λ .

Failure rate	MTTF λ_c	MTTF λ	MTTF λ_h	MTTF λ_1
0.01	10.989	10.999	10.989	19.608
0.02	9.900	9.950	10.869	16.393
0.03	9.009	9.909	10.753	14.084
0.04	8.264	8.564	10.638	12.346
0.05	7.634	7.934	10.526	10.989
0.06	7.092	7.392	10.417	9.901
0.07	6.623	6.823	10.309	9.009
0.08	6.211	6.511	10.204	8.264
0.09	5.848	6.048	10.101	7.634

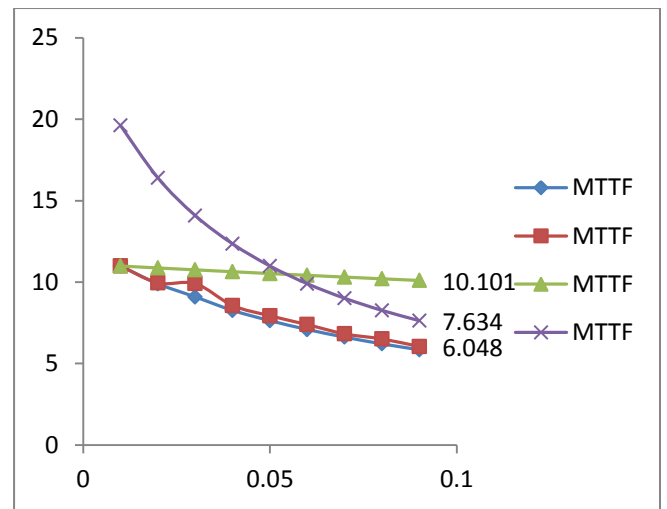


Table and Fig. 2 for failure rate v/s M.T.T.F.

COST ANALYSIS:

Let the service facility be always available, then expected profit during the interval $[0, t)$ is

$$E_p(t) = K_1 \int_0^t P_{up}(t) dt - K_2 t$$

Where K_1 and K_2 are revenue and service cost per unit time.

Hence,

$$E_p(t) = K_1(-0.005516e^{(-2.607t)} + 0.0009037e^{(-1.2227t)} - 0.00002173e^{(-1.0563t)} + 639.21e^{(0.0015579t)} + 639.21 - K_2 t \dots(51)$$

Setting $K_1= 1$ and $K_2= 0.5, 0.4, 0.3, 0.2, 0.1, 0.05,$ and 0.01 respectively and varying $t =0, 10, 20, 30, 40, 50, 60, 70, 80, 90, \dots$ one get Table.3.

Time (t)	$E_p(t); K_2=0.5$	$E_p(t); K_2=0.4$	$E_p(t); K_2=0.3$	$E_p(t); K_2=0.2$	$E_p(t); K_2=0.1$	$E_p(t); K_2=0.05$	$E_p(t); K_2=0.01$
0	0	0	0	0	0	0	0
10	4.877	5.881	6.881	7.881	8.881	9.381	9.781
20	9.605	11.609	13.609	15.609	17.609	18.609	19.409
30	14.183	17.187	20.187	23.187	26.187	27.687	28.887
40	18.613	22.617	26.617	30.617	34.617	36.617	38.217
50	22.897	27.901	32.901	37.901	42.909	45.401	47.401
60	27.037	33.042	39.042	45.042	51.041	54.042	56.442
70	31.036	38.041	45.041	52.041	59.041	62.541	65.341
80	34.896	42.902	50.901	58.901	66.901	70.901	74.101
90	38.619	47.625	56.623	65.625	74.625	79.125	82.724

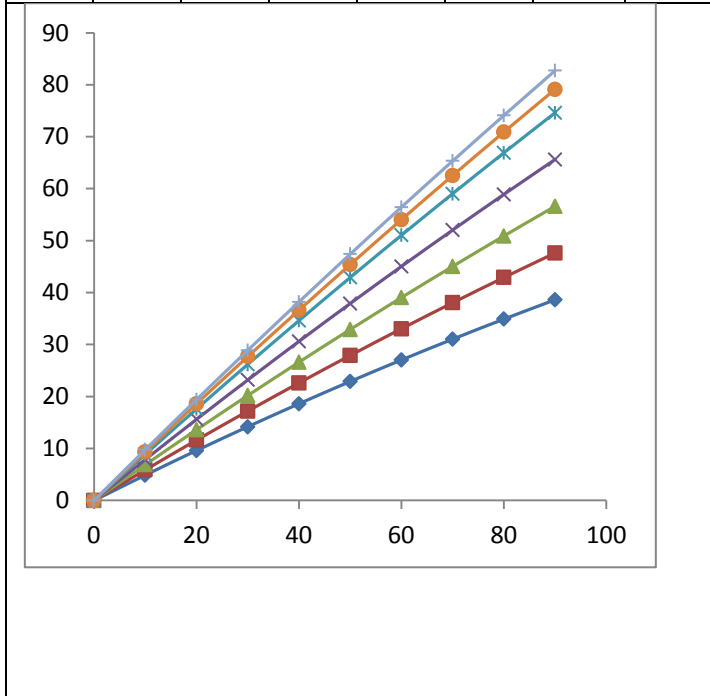


Table and figure for time v/s Expected profit

THE RESULT AND CONCLUSION

Tables 1 and Fig. 1 provide information how availability of the complex repairable system changes with respect to time when failure rates are fixed at different values. When failure rates are fixed at lower values $\lambda_1=0.05, \lambda_2 = 0.06, \lambda_C= 0.01, \lambda = 0.005, \lambda_h= 0.001,$ availability of the system decreases and probability of failure increase, with passage of time and ultimately becomes steady to the value zero after a sufficient long interval of time. Hence one can safely predicts the future behavior of complex system at any time for any given set of parametric values, as is evident by the graphical consideration of the model. Tables 2, and

yield the mean-time-to-failure (MTTF) of the system with respect to variation in $\lambda_1, \lambda_C, \lambda_h$ and λ respectively when other parameters have been taken as constant. When revenue cost per unit time K_1 fixed at 1, service cost $K_2 = 0.5, 0.4, 0.3, 0.2, 0.1, 0.05, 0.01,$ profit has been calculated and results are demonstrated by graphs. One can observed that as service cost decreases profit increases.

The study shows that incorporation of copula improves the reliability of the system significantly.

Acknowledgment

The authors wish to thank Dr.S.B. Singh, Dr. Mangeyram and Professor C.K.Goel for insists us to continuing associate with research work and for helping at varies stages. Dr.Singh have published number of papers in varies repute international journals.

REFERENCES

- (1).A.K. Govil, Operational behaviour of a complex system having shelf-life of the components under preemptive-resume repair discipline, *Microelectronics & Reliab.*,(1974), 13, 97-101.
- (2).F. Lindskog, Modelling dependence with copulas, Risklab report, ETH, Zurich (2000).
- (3).Heinrich Gennheimer, Model Risk in Copula Based Default Pricing Models, Working Paper Series, Working Paper No. 19, Swiss Banking Institute, University of Zurich and NCCR FINRISK (2002).
- (4).Lirong Cui and Haijun Li., Analytical method for reliability and MTTF assessment of coherent systems with dependent components, *Reliability Engineering & System Safety* (2007), 300-307.
- (5).M. R. Melchiori, Which Archimedean copula is right one?,
- (6).P.P. Gupta and M.K. Sharma, Reliability and M.T.T.F evaluation of a two duplex-unit standby system with two types of repair, *MicroelectronReliab.* (1993), 33(3): 291-295.
- (7).R.B. Nelsen, An Introduction to Copulas (2ndedn.) (New York, Springer, 2006).
- (8).Singh, S.B.,Gupta. P.P and Goel.C.K : 'Analytical study of acomplex stands by

redundant systems involving the concept of multifailure-human failure under head-of-line repair policy. Bulletin of pure and applied sciences Vol. of 20 E(No.2) 2001; pp 345-351.

(9).Singh, S.K.; Singh, R.P.; Singh, R.B.: 'Profit evaluation in two unit cold standby system having two type of independent repair facilities', IJOMAS Vol. 8, 277-288 (1992).

(10).Singh,V.V,SinghS.B,Mangeyram,Goel C.K 'Availability analysis of a system having three units super priority , priority and ordinary under pre-emptive resume repair policy' International journal of reliability and applications Vol 11, No1 ,P P (41-53) (2010).,